Salt fingers in double-diffusive systems

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Abstract

“Salt fingering”, the phenomenon of the double-diffusive convection has been studied experimentally. We prepared a system with vertical density variation, which contains salt and sugar solutions, separated one from the other. The “salt fingers” appear at the interface between these layers of salt and sugar solutions. The “salt fingers” have been observed by means of shadow-graph technique.

With the aid of a computer and the video-capture technique, it was possible for the first time to conduct a detailed investigation of the phenomenon. The geometric characteristics of the “salt fingers” such as form, length and width have been investigated.

The way in which these quantities depend on the salinity and sugar concentrations, as well as the thickness of the salt and sugar layers has been verified experimentally. We also verified the predicted form of dependence of the average “salt fingers” width on its vertical length. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

A comparatively recent development in the field of convection is the study of convection in double-diffusive systems, which is represented by two layers with opposite gradients of properties.

For example, a liquid with the opposite concentrations gradient—salinity and temperature—can be considered. It turns out that very surprising convective structures between the double diffusive layers are observed.

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Stommel et al. [1] demonstrated that the structures called “salt fingers” appear in the system, containing hot salt water resting on cold fresh water. The typical “salt finger” works as a pipe pumping fluid up and down. Along these “salt fingers” columns, salty water descends down, intersperses and exchanges heat with a similar array of fresh cold water that is lifted up. The rate of heat exchange exceeds the rate of salt exchange due to the difference between their diffusivities. It is considered to be a very effective mechanism of convection.

The same consideration can be applied, for example, to the system containing salt and sugar, just as in these cases salt plays the role of temperature, and sugar corresponds to salt in the above-mentioned system.

It is the purpose of the present paper to report the result of a series of exploratory laboratory experiments in which the “salt fingers” in the double-diffusive systems have been studied.

In Section 2, we discuss a theory for the formation of the “salt fingers”. The results of the measurements are represented in Section 3 and discussed in Section 4.

2. “Salt fingers” theory

It is well known from experiment that when a liquid placed into gravitational field can be stratified and remains in the equilibrium state, it does not have uniform concentration and density distribution. The density of the liquid increases at the bottom and decreases towards the upper boundary. In any case, in steady state one may observe the following distribution of temperature gradient: a layer of cold water is overlaid by a layer of lighter warm water.

Let us, for instance, imagine the situation when there are both density and temperature gradients and they are both opposite to each other. For example, hot salty water is placed above and fresh cold water below. A question is raised: Can we argue that the hydrostatic stability of this system is still the case? Even if the density distribution corresponds to the steady state situation (the density of the system decreasing upward against gravitational field), the hydrostatic stability is not guaranteed and this was confirmed experimentally.

It was demonstrated in the series of experiments [1–3] conducted at the beginning of the 1960s that a complex structure emerges between two layers belonging to a double-diffusive system. These structures show up like planar polygons, as they seem from above, and columns, as they seem from the side. These structures have come to be known as “salt fingers”. “Salt fingers” represent the sequence of downfalling and uprising water flows, transporting salt and heat (or salt and sugar) via the interface between the layers. The “salt fingers” constitute a very efficient mechanism of diffusion.

It should be pointed out that thermal diffusivity is larger when compared to salt diffusivity: \( K_T/K_S = 100 \). It means that the rate of diffusion for these two components is quite different, but due to double-diffusion convection, the exchange rates of salt and temperature are forced to be comparable.
Let us consider, for example, a small parcel of fresh cold water uprising along a salt finger column. On account of high value of the temperature diffusivity, a very high heat exchange rate exists and this small parcel of water becomes hotter and hotter and quite quickly its temperature and the temperature of ambient environment become equal even during the uprising of the parcel along the “salt-finger” column.

If a liquid parcel from the hot salty layer moves downward, then it cools off very quickly and consequently, the parcel becomes heavier than its environment (just due to a simple reason that it still contains a lot of salt) and the parcel sinks down.

The same considerations can be applied to the salt–sugar double-diffusive system, due to difference in their diffusivities. In this case, the salt solution plays the role of the temperature and sugar corresponds to the salt components of the thermohaline double-diffusive system (salt diffusivity is larger compared to sugar diffusivity).

From a theoretical point of view, in order to describe the observed phenomenon, one can use the basic set of equations: the continuity equation, the Navier–Stokes equation and the diffusion equation. The “salt fingers” cells that appeared at the interface between the two double-diffusive layers can be considered as the instabilities (unstable solutions of this basic set) growing exponentially in time. The Boussinesq approximation [1] is usually applied in order to simplify the equations. In that approximation, the physical quantities as well as $\alpha$—the coefficient of thermal cubical expansion and $\beta$—the coefficient of salinity cubical expansion are considered as constants over definite temperature $T$ and salinity concentration $S$ intervals.

Besides this, $\rho(T,S)$ density of the fluid is represented as a slowly varying function, that depends on $z$ linearly:

$$\rho = \rho_0 \left(1 - \alpha \frac{\partial T}{\partial z} - \beta \frac{\partial S}{\partial z}\right), \quad (2.1)$$

where $\rho_0$ is the density at $z = 0$, $S$ is the salinity, $T$ is the temperature, and $z = x_3$, the coordinates along the $z$-axis.

As a result of these approximations, we have the following set of equations:

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (2.2)$$

$$\frac{D u_i}{D t} = -\frac{\partial}{\partial x_i} \left(\frac{p}{\rho_0} + g z\right) - \alpha(T - T_0) + \beta(S - S_0) + \nu \Delta u_i, \quad (2.3)$$

$$\frac{D T}{D t} = K_t \Delta T, \quad (2.4)$$

$$\frac{D S}{D t} = K_s \Delta S. \quad (2.5)$$

It is the complete set of equations that is required for describing the observed “salt finger” phenomenon across the density interface.

In an effort to accomplish our aim, we should choose appropriate boundary conditions. There are two types of such conditions: the free boundary condition which
means
\[ w|_{z=0,d} = \left. \frac{\partial^2 w}{\partial z^2} \right|_{z=0,d} = 0 \] (2.6)
and the rigid boundary condition which means
\[ w|_{z=0,d} = \left. \frac{\partial w}{\partial z} \right|_{z=0,d} = 0 . \] (2.7)
Here, \( w = u_3 \), \( z \) is the component of the velocity vector field and \( d \) is the distance between the upper and lower planes.

One may select appropriate combinations of these two conditions: (free–free, rigid–rigid, free–rigid). The following system is considered: a fluid is placed between the two planes, the lower plane in our case is the bottom of the beaker, and the upper plane is a free boundary (open air), so we selected the free–rigid boundary condition. In our experiment, the \( S \) liquid represented by sugar solute is placed above and the \( T \) liquid represented by salt solute is placed below.

For the sake of simplicity, the \( T \) and \( S \) fields are treated as linear functions of \( z \) coordinates (only the constant gradients of \( S \) and \( T \) are considered). A Cartesian system of coordinates is the most convenient choice for the description: the origin is placed on the bottom plane and the \( z \)-axis is directed perpendicular to the planes with a positive direction opposite to gravity acceleration vector.

\( T(z) \) and \( S(z) \) can be represented accordingly by the functions
\[
T(z) = T(0)(1 + \alpha z), \quad (2.8)
\]
\[
S(z) = S(0)(1 + \beta z), \quad (2.9)
\]
where \( \alpha \) and \( \beta \) are both constant and positive values.

Now let us investigate a small perturbation of our system that may lead to hydrodynamic instability. Only very small perturbations are considered, which means that all quadratic values (the second order perturbations and other higher order perturbations) are being neglected.

\[
u_i = \tilde{u}_i + u'_i, \quad i = 1, 2, 3, \quad (2.10)
\]
\[ T = \tilde{T} + T', \quad (2.11)\]
\[ S = \tilde{S} + S'. \quad (2.12)\]

The corresponding equations are as follows:
\[
\left( \frac{1}{Pr} \frac{\partial}{\partial t} - \nabla^2 \right) (\nabla \tilde{u}) = -R_T \nabla^2 T' + R_S \nabla^2 S', \quad (2.13)
\]
\[
\left( \frac{\partial}{\partial t} - \nabla^2 \right) \tilde{T} = -w', \quad (2.14)
\]
\[
\left( \frac{\partial}{\partial t} - \tau \nabla^2 \right) \tilde{S} = -w'. \quad (2.15)
\]
Here,

\[ R_T = \frac{g \Delta T d^3}{v K_T} \]  \hspace{1cm} (2.16)

\[ R_S = \frac{g \beta \Delta S d^3}{v K_T} \]  \hspace{1cm} (2.17)

where \( w \) is the fluid velocity along the \( z \) direction, \( R_T \) the Reynolds \( T \) number (for temperature), \( R_S \) the Reynolds number (for salinity) and \( \tau = K_S/K_T \) the ratio of coefficients of salt and temperature diffusivity. \( \tau = 10^{-2} \) for salt–temperature system and \( \tau = \frac{1}{3} \) for salt–sugar system. \( T \) is the average difference of temperature between the two layers, \( S \) the average difference of salinity between the two layers and \( d \) the height of the system.

Then the standard technique, which is usually implemented with an aim to analyze these equations, involves the separation of the variables. We look for the solution of the equations in the form

\begin{align*}
    w' &= \sum_{mkl} R \{ A_w e^{i\lambda t} \sin(\pi mz) e^{-\pi i(kt+ly)} \}, \\
    T' &= \sum_{mkl} R \{ A_T e^{i\lambda t} \sin(\pi mz) e^{-\pi i(kt+ly)} \}, \\
    S' &= \sum_{mkl} R \{ A_S e^{i\lambda t} \sin(\pi mz) e^{-\pi i(kt+ly)} \},
\end{align*}

\hspace{1cm} (2.18) \hspace{1cm} (2.19) \hspace{1cm} (2.20)

\( m, k, l \in \mathbb{Z} \).

Here, \( A_w, A_T, A_S \) are the amplitudes.

This is the method of normal modes, where small perturbations are resolved into modes, which may be considered separately because each of them obeys a set of perturbed equations. The modes represent the development in time of any arbitrary disturbance.

The obtained exponent allows us to analyze the stability of the obtained solutions and estimates the characteristic horizontal and vertical lengths of the “salt fingers”. The first theoretical work, devoted to the study of stability of double-diffusional layer system under the influence of internal gravity waves, was carried out by Stern [4] in 1967.

He analyzed hydrodynamic stability of the “salt fingers” cells and came to the conclusion that “salt fingers” have square planar form with the typical width given by the formula

\[ L = \pi \left[ \frac{g \beta_T z}{v K_T} \right]^{-1/4}, \]  \hspace{1cm} (2.21)

where \( L \) is the horizontal length (width) of the “salt fingers”, \( \beta_T \) the temperature gradient, \( \beta \simeq \Delta T/d \), \( g \) the acceleration due to gravity, \( z \) the temperature expansion coefficient, \( v \) the viscosity and \( K_T \) the temperature diffusivity coefficient.
Based on the obtained relations, one is able to find that the width of the salt fingers is proportional to the length of the salt fingers in \( \frac{1}{4} \) power (scaling relation that was confirmed in numerous experiments).

\[
L \approx h^{1/4}.
\]  

(2.22)

Relying on the above formula, Stern calculated the typical width of the “salt fingers” and compared it with the experimental data obtained by Stommel et al. [1]. Stern found that the width was about 0.2–0.3 cm, very close to the experimental value of the width 0.3 cm.

The planar form of the “salt fingers” cells (a top view) was observed and investigated qualitatively in the first experimental work of Stommel et al. [1] in 1968.

It was discovered that in the salt–temperature double-diffusive system, the planar form of these cells is polygon-like (mainly rectangular or square forms) and the typical width of the “salt finger” cells is about 0.3–0.6 cm.

Shircliff and Turner [5] carried out more detailed quantitative experiments in which the morphology of the “salt fingers” cells was investigated by means of optical methods. It was confirmed that the “salt fingers” have mainly square plane form and the cells oriented both parallel as well as orthogonal to the beaker walls (a rectangular beaker was used). Shircliff and Turner noted that the cells rotated very slowly around their main axes (for each cell, there is a main axis, which is parallel to the \( z \)-axis and passing through the center of the cell).

A more detailed and comprehensive analysis of “salt fingers” regime was performed by Baines and Gill [6]. They drew the state diagram and calculated critical values of the \( R_T, R_S \) for “salt–temperature” and “salt–sugar” double-diffusive systems, for which one can observe “salt fingers” phenomenon, but the mechanism of the “salt fingers” instability was not proposed.

Stern was the first to give a theoretical explanation for the problem in 1967 [1,7], when he wrote down the following condition for the “salt finger” instability:

\[
\frac{\beta F_S}{\kappa v(\partial T/\partial z)} \geq \text{const} \approx 1.
\]  

(2.23)

It follows from the expression that either too larger salinity (S component) flux of gradient of the salt concentration \( F_S \) or too smaller temperature (T components) gradient \( \kappa v(\partial T/\partial z) \) leads to instability, while the internal gravity waves arise and destroy the ends of columnar “salt finger” cells. Relying on this formula, one can explain why the “salt fingers” are destroyed at their ends: the temperature gradients at the ends of the “salt fingers” are too small to be compared with the temperature gradient at the interface between the two layers.

Hence,

\[
\frac{\beta F_S}{\kappa v(\partial T/\partial z)} \geq 1
\]  

(2.24)

is around unity and turbulence convection arises.
Few years later, Stern [8] published another article devoted to the same problem. He suggested that interaction between the “salt fingers” actually results in hydrodynamic instability.

3. Experimental observations of the “salt fingers”

The double-diffusive phenomenon has been thoughtfully investigated in numerous experiments beginning with very simple ones, in which “salt fingers” were explored in a qualitative way, and ending with very sophisticated experiments, in which “salt fingers” were examined quantitatively.

Various flow visualization methods are applied in the investigation of the double-diffusive systems: shadowgraph photography, streak photography may be exploited to make exploratory studies of the motions which can arise as a consequence of opposing vertical gradients [2–4]. Besides that, the elegant optical methods, which provide a means for the “salt finger” morphology exploration on the basis of density as well as refraction index distribution in the double-diffusive system, were employed by Shirtcliffe [5,9–11].

Today, it is possible to use charged coupled devices (CCD camera) to track, record and investigate the double-diffusive systems, as was done in our experiments. We carried out a quantitative experiment by means of optical tools in order to determine the typical characteristics of the “salt fingers” such as the planar form, width, length, area and rate of growth.

In this section, a short outline of the experimental setup is given (see Fig. 1). We took a glass beaker of volume 800 ml (1), a funnel (2), a glass tube (3) and
a clip-regulator (4). The glass tube and the funnel were attached to the holder. The standard scheme consisting of a He–Ne laser (5), a pinhole (6), and a convergent lens (7) was used to create a parallel beam of 2 cm diameter. The beam is declined by the optical mirror (8), so that its direction is changed from horizontal to vertical, and then it enters into the beaker.

The light beam crosses the interface, containing the investigated “salt finger” cells, and then it reflects again from the second optical mirror (9). Now it propagates in the horizontal direction and passes through the second imaging lens (10) with the focus length $F = 5$ cm. The obtained image produced by the lens is recorded by means of the CCD camera.

When the beaker was filled with two solutions, we observed (at the appropriate concentrations of the solutions) a distinct separation between the solutions and the process of the double-diffusive convection, due to the $T–S$ (sugar–salt) instability owing to the different values of the salt and sugar diffusivities. We saw the ink-dyed “salt fingers” cells (see Fig. 2) that descended from the top layer to the bottom layer with

Fig. 2. Shadowgraph of cross section of the salt fingers (top view). The field of view is $10 \times 10$ mm. The cells have polygon-like forms. The salt concentration is 10 g/l and the sugar concentration is 12 g/l.
a speed depending on the concentrations between the mixtures (as well as depending on the system dimensions, diffusion coefficients ratio and form of the boundaries, but these parameters were the same in all our experiments).

The CCD camera records the image and sends it to the computer. This gives us an opportunity to store the received image and process it later. In this way, the main characteristics of the “salt finger” cells such as planar form, area, width as well as the rate of growth were obtained.

In our experiments, the beaker was filled with dyed sugar solution. We usually prepared 200 ml of the sugar solution with various concentrations (from 6 to 19 g/l) and added approximately 8 drops of blue ink (a fluorescent compound can be used as well). The next step was to insert salt solution (200 ml, concentration 10 g/l) below the sugar solution. For the purpose of restricting the fast mixing of the salt and sugar solutions, we poured the salt solution very slowly through the glass tube, which was placed close to the bottom of the beaker. The clip was used in order to regulate the speed at which the salt solution was poured.

4. Results of measurements

The results of our measurements are represented in this section. The experimental data include the shadowgraph images (see Figs. 2 and 3) of the obtained cells and their side view. We investigated the main characteristics of the “salt fingers”: area, type of planar form (the number of vertexes) and the rate of growth at the various salt and sugar concentrations as well as at different volumes of the double-diffusive layers.

Fig. 3. The sequence of shadowgraphs showing the development of the salt fingers section form cells in a typical run recorded at approximately regular intervals of time. The salt concentration is 10 g/l and the sugar concentration is 12 g/l.
Relying on these measurements, we estimated the average width (horizontal length) of the “salt fingers” and compared our experimental data with the theoretical predictions suggested by Turner and Stern [3,12]. The width of the “salt fingers” can be evaluated on the basis of the formula

\[ L \simeq \left( \frac{4K_{t}v}{g} \right) \left( \frac{2ct}{3H(z\Delta T_{0})} \left( 1 + \frac{2ct(\beta \Delta S_{0})^{1/3}}{3H} \right) \right)^{1/4}, \quad (4.1) \]

where \( z \) is the salinity cubic expansion coefficient, \( \beta \) is the salt cubic expansion coefficient, \( K_{t} \) is the salt diffusivity, \( c = 0.1 \text{ cm}^{2}/\text{s} \) is the universal constant according to Turner and Stern [12], \( H \) is the thickness of the double-diffusive layers, \( \Delta S \) is the initial excess of the \( S \) (sugar) component, \( \Delta T \) is the initial excess of the \( T \) (salt) component, \( g \) is the acceleration due to gravity and \( v \) is the kinematic viscosity.

We carried out a series of experiments in order to verify the formula (4.1), which describes the development of the “salt fingers” and found good qualitative correspondence between the prediction and our observations.

In our first series of experiments, the sugar concentration was varied from 6 to 19 g/l, while salt concentration was kept constant at 10 g/l in all these experiments. The beaker was filled with 200 ml of the salt solution and with 200 ml of sugar solution. We determined experimentally the appropriate concentrations of salt and sugar solutions. At the lower limit of the sugar concentration

\[ C_{sug} > 6 \text{ (g/l)} \]

and the upper limit of the sugar concentration

\[ C_{sug} < 20 \text{ (g/l)} \]

one can observe “salt fingering”. When the concentration of the sugar solution was larger than the upper limit or less than the lower limit, we observed an ordinary convection.

We investigated thoroughly the “salt fingers” morphology and concluded that most of the cells are in the form of pentagons and a lesser number of these cells have rectangular form.

After measuring the area of the “salt fingers”, we estimated their average width at various time intervals. Relying on formula (4.1), one can expect that if the sugar concentration increases, then the growth rate of the width of the “salt fingers” increases accordingly. This tendency was observed in our experiment (see Fig. 4). We represent here the experimental data for the case when the sugar concentration was 8, 12 and 16 g/l, while the salt concentration was constant at 10 g/l. As seen from Fig. 4, the theoretical predictions fail to agree with our experiment in a certain case of the sugar concentrations.

In our second series of experiments, we varied the salt concentration from 7 to 14 g/l, while the sugar concentration was kept constant at 12 g/l (see Fig. 5). We found that the “salt fingers” tended to widen faster when the salt concentration was decreased, in line with the theoretical assumption (4.1). The observed correspondence with theory is quite adequate.
Fig. 4. Graphs of the width of the salt fingers versus time at the different sugar concentrations. The salt concentration is fixed at 10 g/l and the sugar concentrations are 8, 12 and 16 g/l. The width of the salt fingers increases faster as the sugar concentration is raised.

Fig. 5. Graphs of the salt fingers width as a function of time at the different salt concentrations. The sugar concentration is 12 g/l and the salt concentrations are 7, 12 and 14 g/l. The salt fingers tend to widen slowly when the salt concentration is increased.
Fig. 6. The measured value of the average width of the salt fingers plotted against time for various thicknesses of the double-diffusive layers. The salt concentration 12 g/l and sugar concentrations 10 g/l remained unchanged, while the thickness of the layers was varied from 2.4 to 4.8 cm. The rate of the salt fingers growth accelerates when the thickness decreases.

The thickness of the double-diffusive layers was altered in our third series of experiments. The salt concentration 12 g/l and sugar concentration 10 g/l remained unchanged, while the thickness of the layers was varied from 2.4 to 4.8 cm (see Fig. 6). The rate of the “salt fingers” growth in the horizontal plane is accelerated when the thickness is decreased, as it can be seen from Fig. 6.

As may be inferred from formula (4.1), one should expect that the lesser the thickness of the double-diffusive layer, the faster will be the growth rate of the “salt fingers” width. The experimental data conform to the theoretical estimation.

In conclusion, we verified the scaling law, that provided the relation between the “salt fingers” length $L$ and width $h$ (see Fig. 7). We represent here the experimental values for the various sugar concentrations—8, 10 and 15 g/l and the constant salt concentration, 12 g/l.

$$L \approx h^{1/4}.$$  \hspace{1cm} (4.2)

On the basis of the obtained data, we concluded that this relation indeed was fulfilled.
5. Conclusions

The “salt-fingering” is one of the most interesting mechanisms of diffusion. It appears at the interface dividing two layers of solutions between which there are gradients of density or temperature, for example, it can be salt and sugar solution or hot salty solution and cold fresh water.

In our project, the selection characteristics of the “salt-fingers” phenomenon were investigated by means of optical tools. We obtained the range of concentrations of salt and sugar as well as the range of layer volumes (or thicknesses) at which the “salt fingers” are observable. The dependence of the average width of the “salt fingers” on salt and sugar concentrations and on the thickness of the layer was measured. The obtained data were compared with the theoretical estimations proposed by Turner and Stern [3,12].

Despite some limitations and drawbacks of our experiments, the results are in good qualitative and sometimes even quantitative agreement with theoretical predictions and with the previous experimental works of many authors.

In particular, we found that “salt fingers” appear only at large values of $R_T$ and $R_S$. It was found that the characteristic time of “salt fingers” growth increases with decreasing sugar concentrations. The scaling law linking the width (the typical horizontal
dimension \( L \) of the “salt fingers”) and the length (the typical vertical dimension \( h \) of the “salt fingers”)

\[
L \approx h^{1/4}
\]

was also verified.

We hope that some further investigations will throw light on this interesting and complicated phenomenon of “salt fingering”.

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